

NAME: \_\_\_\_\_  
(Print) Last Name, First Name

STUDENT NUMBER: \_\_\_\_\_



The Irving K. Barber School of Arts and Sciences

Physics 231— Winter 2013/2014 – Term 1  
FINAL EXAMINATION

Instructor: Jake Bobowski

Saturday, December 5, 2013 Time: 18:00 - 21:00  
Location: ART 386

This Examination was prepared by Jake Bobowski  
Not including this coversheet, the exam consists of 15 numbered pages.

- Attempt all of problems 1 through 4.
- Attempt any three of problems 5 through 8.

If necessary, you may use the backs of pages for calculations.

**You must clearly show your work to receive full credit.**

**Writing down only the correct final answer will not earn full credit.**

**Include units** with the final answer whenever appropriate.

1	2	3	4	5	6	7	8	total
10	10	15	10	10	10	10	10	75

**Free Response:** Write out complete answers to the following questions. Show your work.

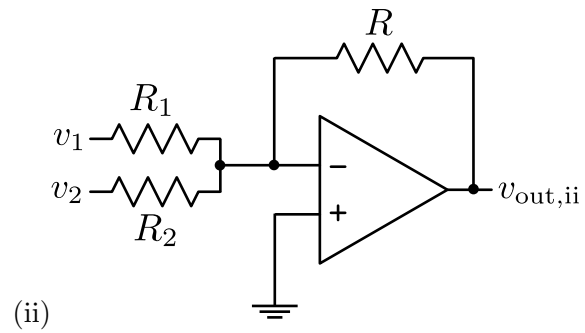
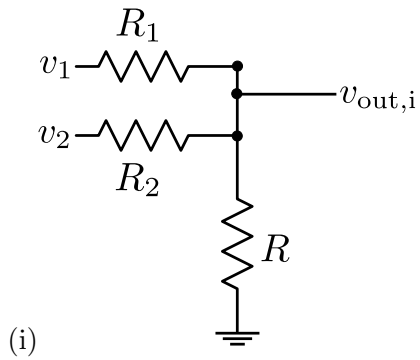
(10<sup>pts</sup>) 1. You have three resistors with specified resistances and uncertainties:  $R_1 \pm \delta R_1$ ,  $R_2 \pm \delta R_2$ , and  $R_3 \pm \delta R_3$ .

(a) If the three resistors are connected in series, what is the equivalent resistance  $R_s \pm \delta R_s$ ? Find expressions for  $R_s$  and  $\delta R_s$  in terms of  $R_1$ ,  $R_2$ ,  $R_3$  and their uncertainties.

(b) If the three resistors are connected in parallel, what is the equivalent resistance  $R_p \pm \delta R_p$ ? Find expressions for  $R_p$  and  $\delta R_p$  in terms of  $R_1$ ,  $R_2$ ,  $R_3$  and their uncertainties.

(c) Suppose you want to make a  $300 \Omega$  resistor. Given the limited equipment that you have in the lab, your options are to combine three  $100 \Omega \pm 5\%$  resistors in series or to combine three  $900 \Omega \pm 5\%$  resistors in parallel. Compare the resulting numerical values of  $\delta R_s$  and  $\delta R_p$ .

(10pts) 2. Consider the following two circuits:

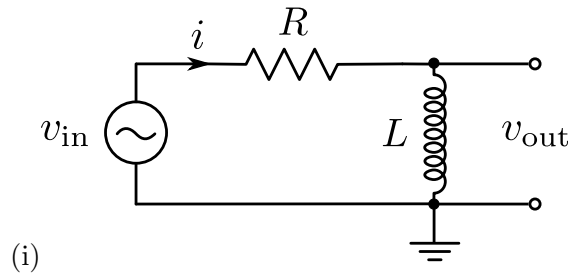


(a) For circuit (i), find an expression for  $v_{out,i}$  in terms of  $v_1$ ,  $v_2$ ,  $R_1$ ,  $R_2$ , and  $R$ .

(b) For circuit (ii), find an expression for  $v_{\text{out,ii}}$  in terms of  $v_1$ ,  $v_2$ ,  $R_1$ ,  $R_2$ , and  $R$ .

(c) Finally, in your expressions for  $v_{\text{out,i}}$  and  $v_{\text{out,ii}}$  from parts (a) and (b) set  $R_1 = R$  and  $R_2 = R$ . Simplify your answers as much as possible and write down the resulting  $v_{\text{out,i}}$  and  $v_{\text{out,ii}}$  expressions.

(15pts) 3. Consider the  $LR$ -series circuit shown below:

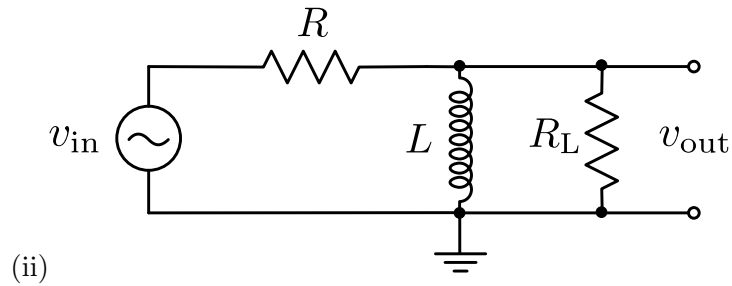


(a) If the input voltage is given by  $v_{in} = V_0 \sin \omega t$ , what are the amplitude  $I_0$  and phase  $\phi$  of the current  $i$ ?

(b) For circuit (i) on the previous page, find an expression for  $\left| \frac{v_{\text{out}}}{v_{\text{in}}} \right|$  in terms of  $\omega$ ,  $R$ , and  $L$ .

Sketch  $\left| \frac{v_{\text{out}}}{v_{\text{in}}} \right|$  as a function of  $\omega$ . What kind of filter is this circuit?

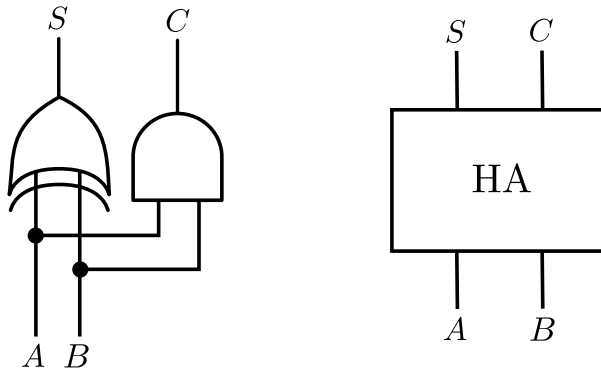
(c) If an oscilloscope is connected across the inductor, the input resistance  $R_L$  of the oscilloscope is placed in parallel with the inductor as shown below in circuit (ii):



For this modified circuit, what is the new expression for  $\left| \frac{v_{out}}{v_{in}} \right|$ ? It is not necessary to do complicated calculations to find the appropriate expression. Instead, try coming up with an equivalent replacement that turns this back into a series circuit similar to the one shown in figure (i).

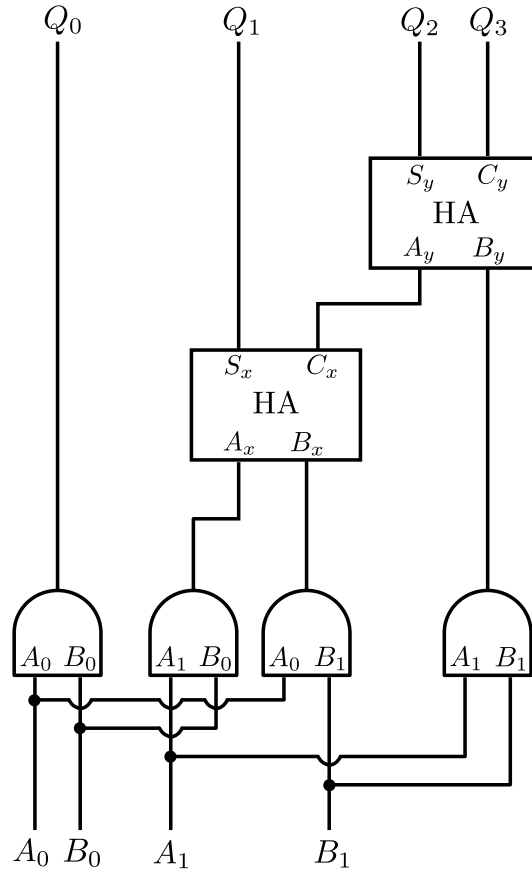


- (10pts) 4. Recall the half-adder circuit used to add to single-bit binary numbers which has two inputs  $A$  and  $B$  and two outputs  $S$  and  $C$ :



- (a) Write down the truth table for the half-adder circuit.

- (b) Fill in the missing data on the truth table given on the next page. What kind of operation is this circuit performing on the pair of 2-bit binary inputs  $(A_1, A_0)$  and  $(B_1, B_0)$ ?



$A_1$	$A_0$	$B_1$	$B_0$	$A_x$	$B_x$	$S_x$	$C_x$	$A_y$	$B_y$	$S_y$	$C_y$	$Q_3$	$Q_2$	$Q_1$	$Q_0$
0	0	0	0												
0	0	0	1												
0	0	1	1												
1	0	0	0												
0	1	0	1												
1	0	1	0												
1	1	1	0												
1	1	1	1												

Complete any of the **three** remaining problems (5, 6, 7, 8).

Clearly indicate which three problems you wish to be graded by entering three numbers into the table below.

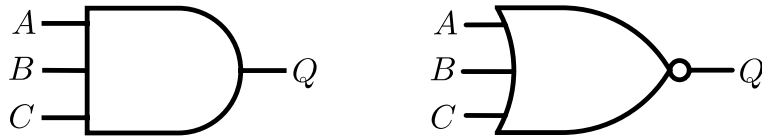
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- (10pts) 5. Use Euler's equation ( $e^{\pm j\phi} = \cos \phi \pm j \sin \phi$ ) to derive the following two trigonometric identities:

$$\cos^2 \phi = \frac{1 + \cos 2\phi}{2}$$

$$\sin^2 \phi = \frac{1 - \cos 2\phi}{2}$$

- (10<sup>pts</sup>) **6.** In class we only talked about logic gates with two inputs, however, it is possible to make some logic gates with any number of inputs. A 3-input gate AND and a 3-input NOR gate are shown below:



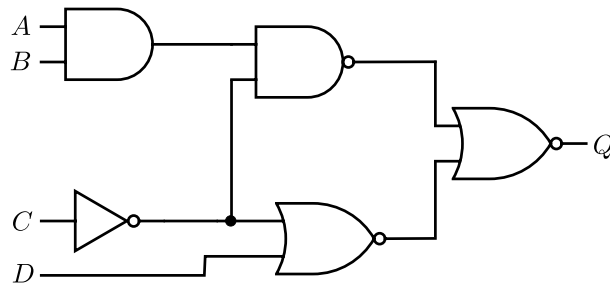
The truth for the 3-input AND gate is:

<i>A</i>	<i>B</i>	<i>C</i>	<i>Q</i>
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

- (a) Write down the truth table for the 3-input NOR gate.

(b) Design a 3-input NOR gate using only transistors and resistors. *Hint*: One common design uses three transistors.

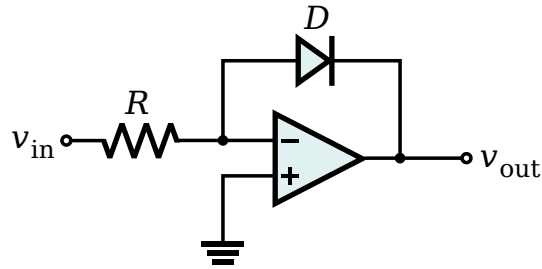
(10pts) 7. Consider the digital circuit shown below:



(a) Write down the logic expression for the output  $Q$ . (For example, recall that the logic expression for the  $X$  AND  $Y$  operation is  $X \cdot Y$ .)

(b) Simplify the expression for  $Q$  obtained in part (a) as much as possible. For part of your solution, you may find De Morgan's theorems helpful:  $\overline{A \cdot B} = \overline{A} + \overline{B}$  and  $\overline{A + B} = \overline{A} \cdot \overline{B}$ . Draw the simplified digital circuit. Are any of the inputs irrelevant to the state of the output  $Q$ ?

- (10pts) 8. Consider the so-called “log amplifier” shown below:



Recall that the current in a diode is given by  $I_D = I_0 (e^{eV_D/k_B T} - 1)$  where  $I_0$  is a constant and  $V_D$  is the voltage across the diode. Assuming that  $v_{in}$  and  $R$  are chosen such that  $I_D \gg I_0$ , show that:

$$v_{out} = G \ln \left( \frac{v_{in}}{RI_0} \right)$$

Find an expression for the proportionality constant  $G$ .